Two Sample T Tests

* When the means of two groups are to be compared (where each group consists of subjects that are not related) then the Excel two-sample t-test procedure is used to perform these calculations.
* NOTE: If your observations are related across “group” as paired or repeated measurements, this in an INCORRECT version of the t-test. For that case, see the tutorial on the Paired t-test.

**Assumptions:**

* Subjects are randomly assigned to one of two groups.
* The distribution of the means by group is normal with equal variances.
* Sample sizes between groups do not have to be equal.

**Test:**

* The hypotheses for the comparison of means from two independent groups are:

**Ho: m1 = m2** (means of the two groups are equal)
**Ha: m1 ¹ m2** (means are not equal)

* The test statistic is a student’s t-test with N‑2 degrees of freedom, where N is the total number of subjects.
* A low p‑value indicates evidence to reject the null hypothesis in favor of the alternative. In other words, there is evidence that the means are not equal.
* For example, suppose we are interested in comparing SCORES across GROUPS, where there are two groups. The purpose is to determine if the mean SCORE on a test is different for the two groups tested (i.e., control and treatment groups). The example data is shown here:

|  |  |
| --- | --- |
| Group | Scores |
| 1 | 20 |
| 1 | 23 |
| 1 | 32 |
| 1 | 24 |
| 1 | 25 |
| 1 | 28 |
| 1 | 27.5 |
| 2 | 25 |
| 2 | 46 |
| 2 | 56 |
| 2 | 45 |
| 2 | 46 |
| 2 | 51 |
| 2 | 34 |
| 2 | 47.5 |

* In this example, GROUP contains two values, 1 or 2, indicating which group each subject was in. The t-test will be performed on the values in the variable (column) named SCORE.

**An independent group t-test is done in two steps:**

**Step 1:**

* Decide if the variances are equal in both groups, which determines the type of t-test to perform (one that assumes equal variances or one that doesn’t make that assumption.)
* A conservative approach suggested in some texts is to always assume unequal variances.
* Another approach is to do a statistical test to determine equality.

**Step 2:**

* Depending on your decision about the equality of variances you either perform the version of the t-test that assumes equality of variances or other one that doesn’t make that assumption.

**Determine Equality of Variance**

* If you take the conservative approach, skip this test and proceed to the version of the t-test that does not assume equality of variance.
* To do a statistical test to determine equality of variance, follow these instructions: (The test for equality of variances is an F-test.)
1. In Excel, select Tools/ Data Analysis / F-Test Two Sample for Variance.
2. In the F-Test Two Sample for Variance dialog box:
* For the Input Range for Variable 1, highlight the seven values of Score in group 1 (values from 20 to 27.5).
* For the input range for Variable 2, highlight the eight values of Score in group 2 (values from 25 to 47.5).
* Leave the other items at their default selections. This dialog box is shown below. Click OK.



1. The following results are produced by Excel:

|  |  |  |
| --- | --- | --- |
| F-Test Two-Sample for Variances |  |  |
|  |  |  |
|  | Variable 1 | Variable 2 |
| Mean | 25.64285714 | 43.8125 |
| Variance | 15.22619048 | 96.42410714 |
| Observations | 7 | 8 |
| df | 6 | 7 |
| F | 0.157908545 |  |
| P(F<=f) one-tail | 0.019378053 |  |
| F Critical one-tail | 0.23771837 |  |

* Notice the highlighted probability p=0.01937. This is a one-tail p-value associated with the test for equality of variance. Generally, if this value is less than 0.05 you assume that the variances are NOT equal.
1. If the variances are assumed to NOT be equal, proceed with the t-test that assumes non-equal variances.
2. If the variances are assumed to be equal, proceed with the t-test that assumes equal variances.

**Perform the t-test**

* The process of doing the t-test in Excel is similar for both the equal and unequal variances case – the main difference is which version you select from the menu.  Suppose you select the unequal version of the two-sample t-test – this is how you proceed:
1. Select Tools/ Data Analysis/ t-Test: Two Sample assuming Unequal Variances
2. For the Input Range for Variable 1, highlight the seven values of Score in group 1 (values from 20 to 27.5). For the input range for Variable 2, highlight the eight values of Score in group 2 (values from 25 to 47.5). Leave the other items at their default selections.
* This dialog box is shown below. Click OK.



1. The following output is created:

|  |  |  |
| --- | --- | --- |
| t-Test: Two-Sample Assuming Unequal Variances |   |   |
|   |   |   |
|   | Variable 1 | Variable 2 |
| Mean | 25.64285714 | 43.8125 |
| Variance | 15.22619048 | 96.42410714 |
| Observations | 7 | 8 |
| Hypothesized Mean Difference | 0 |   |
| Df | 9 |   |
| t Stat | -4.816944724 |   |
| P(T<=t) one-tail | 0.000475506 |   |
| t Critical one-tail | 1.833112923 |   |
| P(T<=t) two-tail | 0.000951012 |   |
| t Critical two-tail | 2.262157158 |   |

* Notice that the two sample mean values (variance) are 25.64(15.23) and 43.81(96.42). The two tailed calculated t-statistic is 4.82 and the highlighted p-value for this test is p=0.001. (0.000951012). Since the p-value is less than 0.05, this provides evidence to reject the null hypothesis of equal means.

**As an example of how this might be reported in a journal article:**

**Methods:**

* A preliminary test for the equality of variances indicates that the variances of the two groups were significantly different F=.157, p=.02.
* Therefore, a two-sample t-test was performed that does not assume equal variances.

**Results:**

* The mean score for group 1 (M=25.64 SD= 3.9021, N= 7) was significantly smaller than the scores for group 2 (M=42.81, SD=9.82, N= 8.) using the two-sample t-test for unequal variances, t(9) = -4.82, p <= 0.001. (Technically, the degrees of freedom for this unequal variances t-test should be 9.4 instead of 9, but Excel unfortunately rounds off the DF, so it is reported incorrectly. Years ago, it used to be conventional to round down if you were constructing a table for a probability level, but most statistical programs now calculate the correct p-value using a fractional DF through interpolation.)
* Notice that the variance is reported rather than the standard deviation, as shown in the Excel results table. You can calculate the standard deviation using Tools/ Data Analysis / Descriptive statistics.
* When the variances are assumed equal, the analysis is similar, select Tools/ Data Analysis/ t-Test: Two Sample assuming Equal Variances

**One Sample T-Test**

* Because Excel doesn’t have a function for one sample T-Tests, you can manipulate the program by creating the second sample which would be the population mean that you are testing.
* A one sample T-Tests looks to see if the sample mean is equal to the population mean.
* You are only given one sample, and then the send second data set would be the population mean that you chose.

**Two-Tail Test**

* Two-tailed test allots half of your alpha to testing the statistical significance in one direction and half of your alpha to testing statistical significance in the other direction. This means that .025 is in each tail of the distribution of your test statistic.
* When using a two-tailed test, regardless of the direction of the relationship you hypothesize, you are testing for the possibility of the relationship in both directions.
* For example, we may wish to compare the mean of a sample to a given value x using a t-test.  Our null hypothesis is that the mean is equal to x. A two-tailed test will test both if the mean is significantly greater than x and if the mean significantly less than x. The mean is considered significantly different from x if the test statistic is in the top 2.5% or bottom 2.5% of its probability distribution, resulting in a p-value less than 0.05.

**One-Tailed Tests**

* If you are using a significance level of .05, a one-tailed test allots all of your alpha to testing the statistical significance in the one direction of interest.  This means that .05 is in one tail of the distribution of your test statistic.
* When using a one-tailed test, you are testing for the possibility of the relationship in one direction and completely disregarding the possibility of a relationship in the other direction.
* Let's return to our example comparing the mean of a sample to a given value x using a t-test.  Our null hypothesis is that the mean is equal to x. A one-tailed test will test either if the mean is significantly greater than x or if the mean is significantly less than x, but not both. Then, depending on the chosen tail, the mean is significantly greater than or less than x if the test statistic is in the top 5% of its probability distribution or bottom 5% of its probability distribution, resulting in a p-value less than 0.05.  The one-tailed test provides more power to detect an effect in one direction by not testing the effect in the other direction.

**References:**

<http://www.stattutorials.com/>